Interpolants in Two-Player Games

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We present a new application of interpolants in the context of two-player games. Two-player games is a useful formalism for the synthesis of reactive systems, with applications in device driver development, hardware design, industrial automation, etc. In particular, we consider reachability games over a finite state space, where the first player (the controller) must force the game into a given goal region given any valid behaviour of the second player (the environment). A *winning strategy* for the controller is a mapping that associates with every state a controllable action to play in this state. To solve a game, the algorithm must (1) prove that there exists a winning strategy for the controller and (2) generate a concrete winning strategy. In this work we focus on the latter problem, which we refer to as *strategy extraction*.

We address the strategy extraction problem in the context of a new counterexample-guided SAT-based algorithm for solving reachability games, recently proposed by Narodytska et al. [1]. If a strategy for the controller exists, the algorithm produces a certificate of strategy existence in the form of a game tree that specifies a set of moves of the controller at each round of the game. Figure 1 shows an example game tree produced by the algorithm. Each node of the tree corresponds to a set of states and each edge of the tree is labeled with a move of the controller. The root node is labeled with a set of initial states, $\{1, 2, 3, 4\}$, and its outgoing edges are labeled with possible moves for the controller in the first round of the game (a, b or c). Note that the game tree does *not* specify which move the controller should play in each state. The certificate only guaranties that playing one of these moves out of the entire universe of possible moves is sufficient to win the game.

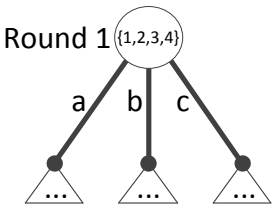

Figure 1: A game tree.

We propose an interpolation-based technique to extract a concrete winning strategy from the game tree. We construct a CNF formula in each leaf of the game tree that encodes a run of the game through this leaf that is winning for the environment. By the construction of the game solving algorithm, we know that conjunction of these CNFs is unsatisfiable. We obtain a resolution refutation from this unsatisfiable formula and use interpolants to determine a concrete strategy in the root. Then we proceed to the next node in the game tree.

References

[1] Narodytska, N., Legg, A., Bacchus, F., Ryzhyk, L., Walker, A.: Solving Games without Computing Controllable Predecessors. In: Proceedings of CAV'14(2014).

*NICTA is funded by the Australian Government through the Department of Communications and the Australian Research Council through the ICT Centre of Excellence Program.