DEPLETION-REGION RECOMBINATION IN SILICON SOLAR CELLS: WHEN DOES $m_{DR} = 2$?

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ABSTRACT: This paper examines the ideality factor of depletion-region recombination m_{DR}, with a particular emphasis on its maximum value. Several theoretical models of depletion-region recombination are discussed and it is shown that the models with more assumptions tend to overestimate m_{DR}. Numerical simulations are then used to determine the maximum value of m_{DR} for both step-junction and diffused-junction solar cells, for the case when the trap density is uniformly distributed across the depletion region. The maximum value of m_{DR} is found to increase with doping from 1.7 to 2 for step junctions; and to be approximately 1.8 for all practical doping levels of diffused junctions.
Keywords: Recombination - 1; Simulation - 2; Modelling - 3.

1. INTRODUCTION

The recombination that occurs in the depletion region of a solar cell can have a significant influence on the solar cell's efficiency. To quantify this influence, the depletion-region recombination current is often taken to be proportional to $\exp(qV_j/m_{DR}kT)$, where m_{DR} is the ideality factor of the depletion-region recombination. m_{DR} distinguishes depletion-region recombination from most other sources of recombination, which have an $\exp(qV_j/kT)$ dependence on the junction voltage V_j.

To model or characterise the influence of depletion-region recombination in a solar cells requires the magnitude of m_{DR}. Several theoretical models indicate that m_{DR} takes a value between 1 and 2, where the exact value of m_{DR} depends on conditions such as the number of traps in the "forbidden gap", the doping density, and the junction voltage. While the theoretical models predict similar trends for how m_{DR} depends on these conditions, they also predict different values for the actual magnitude of m_{DR}.

This paper investigates the discrepency between the theoretical models of depletion-region recombination, and compares the value of m_{DR} predicted by the models to that obtained by numerical simulations. Numerical simulations are then used to determine values of m_{DR} for conditions that are relevent to silicon solar cells, such as diffused asymmetrical p–n junctions (e.g. for bulk-crystalline solar cells) and step p–n junctions (e.g. for thin-film solar cells). Particular attention is paid to the maximum value that m_{DR} can take under a given set of conditions.

2. GENERAL THEORY OF DEPLETION-REGION RECOMBINATION

The general theory of depletion-region recombination is described briefly to provide a qualitative understanding of the factors that influence m_{DR}.

The fundamental and defining difference between depletion-region recombination and other sources of recombination is related to the rapid variation of the carrier concentrations that occurs across a p–n junction. This rapid variation can be seen in Fig. 1, which plots the electron $n(x)$ and hole $p(x)$ concentration

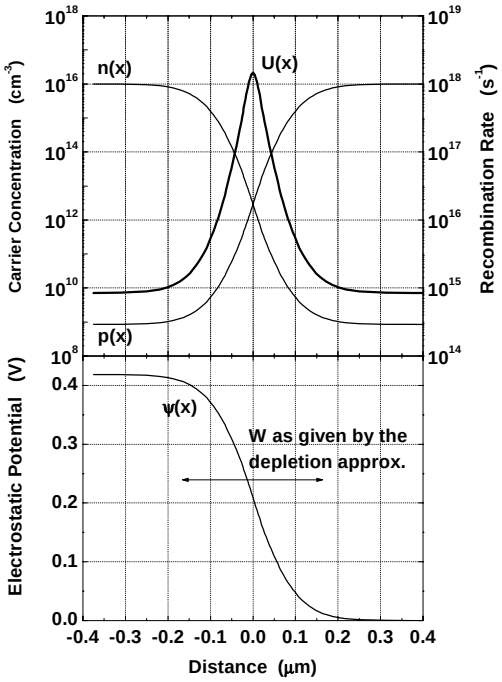

Figure 1: Numerical solution of $n(x)$, $p(x)$, $U(x)$ and $\psi(x)$ for a symmetrical step junction of doping concentration 10^{16} cm^{-3} under special-case conditions.

across a symmetrical step junction, and the subsequent variation of the electrostatic potential $\psi(x)$ (at $V_j = 0.3$ V).

Fig. 1 also plots the recombination rate $U(x)$ in the depletion region. Since at low-to-medium voltages, the dominant source of recombination in a silicon solar cell is due to Shockley–Read–Hall (SRH) recombination, $U(x)$ was determined with the SRH equation:

$$U(x) = \frac{n(x)p(x) - n_i^2}{\tau_{p0}(x)[n(x) + n_1(x)] + \tau_{n0}(x)[p(x) + p_1(x)]}, \quad (1)$$

which assumes that there is a single trap at an energy level E_t that lies between the conduction and valence bands; $\tau_{n0}(x)$ and $\tau_{p0}(x)$ are the electron and hole lifetime coefficients, $n_1(x)$ and $p_1(x)$ are the electron and

hole concentrations when E_t coincides with the Fermi level, and n_i is the intrinsic carrier concentration.

For the plot of Fig. 1, $U(x)$ was determined by imposing several conditions that simplify the analysis and lead to a maximum value of m_{DR}. (As described in Section 3, some of the theoretical models are restricted to these "special-case" conditions). The conditions are that the junction voltage is sufficiently large (0.3 V) to ensure that $n(x)p(x) \gg n_i^2$; E_t is equal to the intrinsic Fermi level E_i throughout the depletion region, giving $n_1(x) = p_1(x) = 0$; and the minority carrier lifetime coefficients are equal and uniform throughout the depletion region, giving $\tau_{n0}(x) = \tau_{p0}(x) = \tau_0$. With these restrictions, Eq. 1 simplifies to

$$U(x) = \frac{n(x)p(x)}{\tau_0[n(x) + p(x)]}, \qquad (2)$$

from which it is evident that $U(x)$ is strongly peaked and centered at the point where $n(x) = p(x)$, defined here to be $x = 0$ (Fig. 1).

An estimate for m_{DR} can be attained from Eq. 2 if it is assumed that the quasi-Fermi levels are constant across the depletion region and separated by V_j, that is, $pn = n_i^2 \exp(qV_j/kT)$ [4]. Hence, at $x = 0$, $n(0) = p(0) = n_i \exp(qV_j/2kT)$, and Eq. 2 simplifies further to $U(0) = n_i \exp(qV_j/2kT)/\tau_0$. Thus, with the special-case conditions, $U(x)$ is a maximum at $x = 0$ and is proportional to $\exp(qV_j/2kT)$. If the depletion-region recombination current J_{DR} were due solely to the recombination that occurs at $x = 0$, m_{DR} would equal 2.

But recombination from other locations in the depletion region also contributes to J_{DR}. Far from the centre of the depletion region, $U(x)$ becomes like bulk SRH recombination and is proportional to $\exp(qV_j/kT)$. (For example, for $p(x) \gg n(x)$, Eq. 2 is simplified to $U(x) = n(x)/\tau_0$, and since as a majority carrier, $p(x)$ depends weakly on V_j, $n(x)$ and $U(x)$ are proportional to $\exp(qV_j/kT)$.) Thus, if J_{DR} were due solely to the recombination that occurs at large $|x|$, m_{DR} would equal 1.

From the above discussion, it is evident that as $|x|$ increases, $U(x)$ changes from being proportional to $\exp(qV_j/2kT)$ to being proportional to $\exp(qV_j/kT)$. This change occurs due to the increasing difference between $n(x)$ and $p(x)$. Since J_{DR} is the integration of $U(x)$ across the width of the depletion region, m_{DR} takes a value between 1 and 2.

Further to this discussion on m_{DR}, it is significant that the above discussion is based on the special-case conditions, which cause $U(x)$ to be strongly peaked at $x = 0$. This gives rise to a larger value of m_{DR} (but that is still less than 2). In almost all cases, variations from these conditions lead to a reduction in the value of m_{DR}. Such variation include when $E_t \neq E_i$ [1][2], and when there is more than one trap level [2][3]. The value of m_{DR} is increased if $\tau_{n0}(x)$ and $\tau_{p0}(x)$ are smaller near $x = 0$ than elsewhere in the depletion region, since $U(x)$ would become more strongly peaked about $x = 0$. This latter situation is discussed further in Section 4.

In summary, m_{DR} takes a value between 1 and 2 but $m_{DR} \sim 2$ only under very specific conditions that lead to the recombination being very strongly peaked about the point where $n(x) = p(x)$.

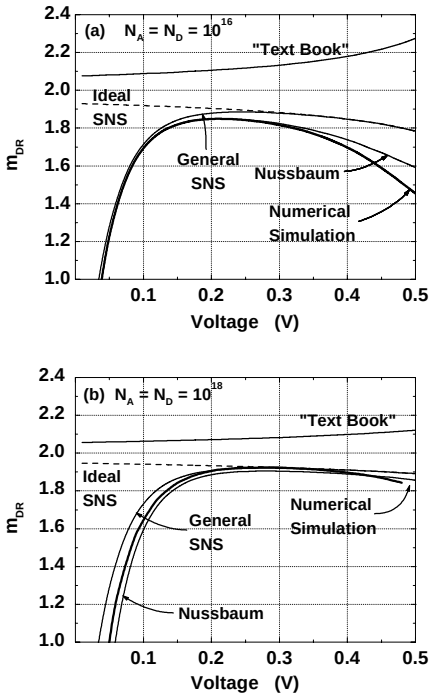

Figure 2: m_{DR} vs V_j as predicted by the theoretical models and by numerical simulation for the special-case conditions—see text.

3. COMPARISON OF MODELS FOR DEPLETION-REGION RECOMBINATION

There are several theoretical models of depletion-region recombination that can be used to determine a specific value of m_{DR}. Four of these models are compared in this section; they are, a simple model found in some text books (e.g., [4][5]), the ideal model of Sah, Noyce and Shockley (SNS) [1]; the general model of SNS [1]; and the model of Nussbaum [6].

Fig. 2 plots the value of m_{DR} that is predicted by each of the theoretical models, for the same special-case conditions used in Fig. 1. The numerical solution is also plotted for comparison. The figure indicates that there is some discrepency between the models. Following a summary of the assumptions entailed in the models, the reasons for the discrepency are described. (These and other models have also been compared with regards to J_{DR} [9]).

3.1 Assumptions common to all theoretical models

To determine J_{DR}, each of the theoretical models follow a similar derivation. First it is assumed that the current flow across the junction is one-dimensional, so that J_{DR} is given by the integration of $U(x)$ across the width of the depletion region W, $J_{DR} = q\int_W U(x)dx$.

To determine $U(x)$, it is assumed the recombination rate follows the SRH formula, which requires that there be a single trap at an energy E_t, and that the semiconductor is non-degenerate. It is further assumed that the traps are uniformly distributed and the lifetime coefficients are constant throughout the depletion region; thus from Eq. 1, $U(x)$ varies only with $n(x)$ and $p(x)$.

$n(x)$ and $p(x)$ are determined by assuming that

$'$ are related to $\psi(x)$ following Boltzmann statistics and that the electron and hole quasi-Fermi levels are constant and separated V_j [1].

Finally, by determining an expression for $\psi(x)$, J_{DR} can be found. To simplify the expression for $\psi(x)$, all of the models assume that the junction is a step junction.

Once J_{DR} is known, m_{DR} can be determined from $m_{DR} = \frac{q}{KT}\left(\frac{dV_j}{d\ln J_{DR}}\right)$ [3].

3.2 "Text book" model

A "text-book" model provides the simplest derivation of J_{DR} (e.g., [4][5]). It is restricted to the special-case conditions that lead to Eq. 2 (Section 2). The model assumes that $U(x)$ is constant across the depletion region and equal to $U(0)$. The integration of $U(x)$ across W is then trivial and given by $J_{DR} = qWU(0)$. It can be seen in Fig. 1, that the assumption, $U(x) = U(0)$ leads to an overestimation of J_{DR}.

At first glance, the "text book" model appears to show that $m_{DR} = 2$, since $U(0) \propto \exp(qV_j/2kT)$. However, W is not constant with V_j; from the depletion approximation, $W \propto \sqrt{V_{bi} - V_j}$ [4]. It follows that $m_{DR} = 2$ only when $V_{bi} \gg V_j$, but this situation does not arise for practical silicon solar cells. Thus the "text-book" model predicts $m_{DR} > 2$, contradicting the qualatitive discussion of the preceding section. Fig. 2 shows the value of m_{DR} as a function of V_j for (a) $V_{bi} = 0.71$ V and (b) $V_{bi} = 0.95$ V, and indicates that the model overestimates m_{DR}.

3.3 Ideal SNS model

With the first paper on the subject, Sah, Noyce and Shockley (SNS) presented an idealised model that provides a good intuitive description of depletion-region recombination [1]. (Nussbaum provides an alternative derivation of the same model [6]). Like the "text book" model, the ideal SNS model is restricted to the special-case conditions that lead to Eq. 2. But rather than setting $U(x) = U(0)$, this model determines $U(x)$ by assuming that the potential gradient $\psi'(x)$ is constant throughout the depletion region and equal to $(V_{bi} - V_j)/W$, where W is again defined by the depletion approximation. Fig. 1 indicates that this assumption becomes increasingly less valid with increasing $|x|$, as $\psi'(x)$ actually decreases and approaches zero. Consequently, the model overestimates J_{DR}, though to a lesser extent than the "text book" model.

The subsequent integration of $U(x)$ across W yields $J_{DR} \propto U_{\max}/\sqrt{V_{bi} - V_j}$ [1][6]. Similar to the "text-book" model, it follows that $m_{DR} = 2$ only when $V_{bi} \gg V_j$; otherwise, $m_{DR} < 2$. This result indicates that by taking into account the variation of W with respect to V_j, and the recombination that occurs where $n(x)$ and $p(x)$ are not equal, m_{DR} must take a value that is less than 2. As seen in Fig. 2, the ideal SNS model slightly overestimates m_{DR}.

3.4 General SNS model

SNS also provide a more general model of depletion-region recombination for symmetrical step junctions, that is not restricted to the special-case conditions described above [1]. (The model is extended by Choo to include asymmetrical step junctions [8]). Like their ideal model, their general model also makes the assumption that $\psi'(x)$ is constant, and subsequently overestimates both J_{DR} and m_{DR}.

As seen in Fig. 2, the general SNS model differs from the ideal SNS model only at low voltages. This difference arises from the simplification in the ideal SNS theory that $p(x)n(x) \gg n_i^2$.

3.5 Nussbaum model

The most complex and the most accurate of the theoretical models is that presented by Nussbaum [6][7]. Like the general SNS theory, Nussbaum's model is not limited to the special-case conditions, but it differs from the SNS theory in two ways: firstly, $\psi(x)$ is determined with Poisson's equation; and secondly, the depletion approximation is not used to determine W, but rather, the limits of the integral are defined as where $\psi'(x) = 0$. (These integral limits can be redefined to conform with W as calculated by the depletion approximation [9].) Fig. 2 demonstrates that of the four models, the Nussbaum model most closely matches the numerical simulation.

3.6 Numerical simulation

The semiconductor device simulator, DESSIS [10], was also used to determine m_{DR}. DESSIS solves the fully coupled set of semiconductor differential equations, and does not require any of the afore-mentioned assumptions. Consequently, when appropriately applied, DESSIS can be used to determine the most accurate value of m_{DR}.

To provide a meaningful comparison with the theoretical models, the DESSIS model was constructed so that the carrier flow was essentially one-dimensional, and so that $U(x)$ follows the SRH formula (Eq. 1), where $n_1(x) = p_1(x) = 0$, and $\tau_{n0}(x) = \tau_{p0}(x) = \tau_0$. The numerical solutions are shown in Fig. 2.

Note that the numerical solution gives the total recombination current and not J_{DR}. To determine J_{DR} (and hence m_{DR}), the diffusion recombination current, which is proportional to $\exp(qV_j/kT)$, was subtracted from the total recombination current. In most cases, however, this subtraction was insignificant in the voltage range of interest (0.2–0.4 V), since at these voltages, J_{DR} was the dominant source of the recombination current.

4. MAXIMUM m_{DR} for SILICON SOLAR CELLS

DESSIS was used to determine the maximum value of m_{DR} for silicon solar cells, for the case where $\tau_{n0}(x)$ and $\tau_{p0}(x)$ are uniformly distributed across the depletion region; this value occurs when the special- case conditions are applied (Section 2). As well as providing the most accurate estimate of m_{DR}, the numerical solutions can be applied to diffused junctions.

For the special-case conditions, m_{DR} is approximately constant with V_j in the range 0.2–0.4 V (Fig. 2). For silicon solar cells, this voltage range is the most appropriate for an investigation into depletion-region recombination, since at lower voltages, $n(x)p(x) \not\ll n_i^2$, and at higher voltages, other sources of recombiantion contribute significantly to the total recombination current. In the results that follow, the average value of m_{DR} in this voltage range and is shown as the thick

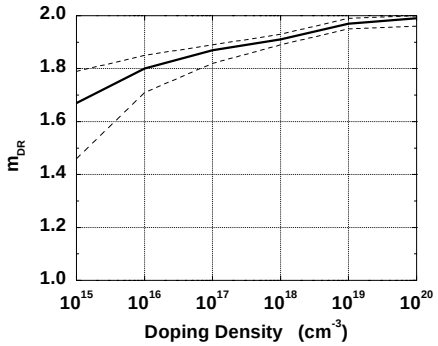

Figure 3: Numerical simulation for the maximum m_{DR} for a silicon step junction, where τ_{n0} and τ_{p0} are uniform across the depletion region. Bold line shows the average value and the dashed lines show the limits of m_{DR} between 0.2–0.4 V.

line in Figs. 3 and 4. The dashed lines indicate the highest and lowest value of m_{DR} in the voltage range.

Fig. 3 plots the maximum value of m_{DR} against the doping density for a symmetrical step junction, when τ_{n0} and τ_{p0} are constant with x. It indicates that for very heavy doping, m_{DR} approaches 2. This result arises from $U(x)$ being extremely peaked about the p–n junction. For lighter doping, m_{DR} decreases to ~ 1.7. To a good approximation, it was found that m_{DR} takes the value of the more lightly doped side of an asymmetrical step junction, and that m_{DR} is independent of τ_{n0} and τ_{p0}.

Fig. 4 plots m_{DR} against the doping density for a diffused n^+–p junction, when τ_{n0} and τ_{p0} are constant with x. It indicates that for all practical doping levels, the maximum value of m_{DR} is ~ 1.8. To a good approximation, it was found that m_{DR} is independent of τ_{n0} and τ_{p0}.

Note that the values plotted in Figs. 3 and 4 could be exceeded if $\tau_{n0}(x)$ and $\tau_{p0}(x)$ were smaller at $x = 0$ than elsewhere in the depletion region (since this would cause $U(x)$ to be more strongly peaked where $n(x) = p(x)$. Such a situation might arise for a grown step junction where there is a greater number of defects at the boundary between n-type and p-type silicon. The situation is less likely to occur in diffused silicon junctions.

In summary, if a solar cell exhibits an m_{DR} that is greater than that plotted in Fig. 3 (for a given doping level), it can be concluded that either the trap density near the location where $n(x) = p(x)$ is greater than elsewhere, or the recombination mechanism does not follow the SRH equation.

5. CONCLUSION

This paper examined the value of m_{DR}, with a particular emphasis on its maximum value. It was demonstrated that there is a discrepency between several theoretical models of depletion-region recombination; the models that contain more assumptions tend to overestimate m_{DR}. Numerical simulations, which make fewer assumptions than the theoretical models, were then used to determine the maximum value of m_{DR} for both step-junction and diffused-junction silicon solar

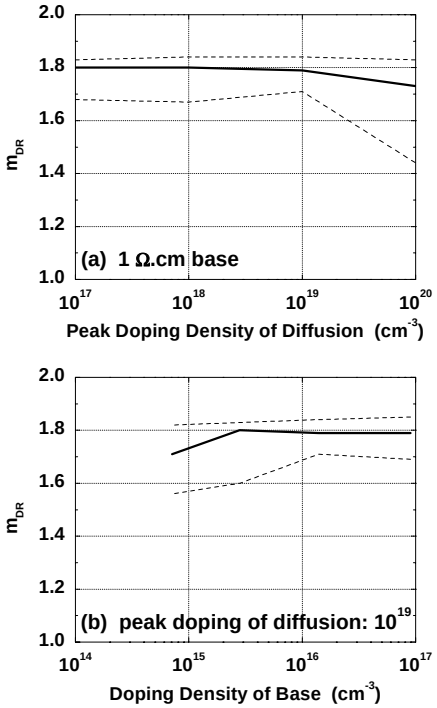

Figure 4: Numerical simulation for the maximum m_{DR} for a silicon diffused junction, where τ_{n0} and τ_{p0} are uniform across the depletion region. Bold line shows the average value and the dashed lines show the limits of m_{DR} between 0.2–0.4 V.

cells, for the case when the trap density is uniformly distributed across the depletion region. The maximum value of m_{DR} was found to increase with doping from 1.7 to 2 for step junctions; and to be approximately 1.8 for all practical doping levels of diffused junctions.

It is concluded from this study that for silicon solar cells that exhibit a strong $\exp(qV_j/2kT)$ recombination current, this recombination current can be explained as a consequence of depletion-region recombination only if the solar cell conatains a very heavily doped step junction, or if it contains a greater denisty of traps at the centre of the depletion region than elsewhere in the depletion region.

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